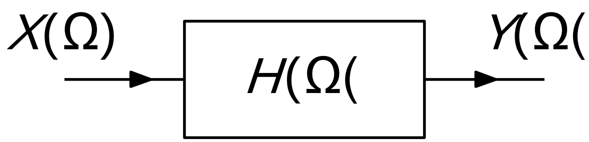
- Since the frequency response H is simply the frequency spectrum of the impulse response h, if h is *real*, then

$$|H(\Omega)| = |H(-\Omega)| \quad \text{and} \quad \arg H(\Omega) = -\arg H(-\Omega($$

)i.e., the magnitude response $|H(\Omega)|$ is *even* and the phase response $\arg H(\Omega)$ is *odd*.(

- Consider a LTI system with input x, output y, and impulse response h, and let X, Y, and H denote the Fourier transforms of x, y, and h, respectively.
- Often, it is convenient to represent such a system in block diagram form in the frequency domain as shown below.



- Since a LTI system is completely characterized by its frequency response, we typically label the system with this quantity.

- Many LTI systems of practical interest can be represented using an *Nth-order linear difference equation with constant coefficients*.
- Consider a system with input x and output y that is characterized by an equation of the form

$$\sum_{k=0}^{N} b_k y(n-k) = \sum_{k=0}^{M} a_k x(n-k).$$

- Let h denote the impulse response of the system, and let X, Y, and H denote the Fourier transforms of x, y, and h, respectively.
- One can show that $H(\Omega)$ is given by

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{\sum_{k=0}^{M} a_k (e^{j\Omega})^{-k}}{\sum_{k=0}^{N} b_k (e^{j\Omega})^{-k}} = \frac{\sum_{k=0}^{M} a_k e^{-jk\Omega}}{\sum_{k=0}^{N} b_k e^{-jk\Omega}}.$$

- Each of the numerator and denominator of H is a *polynomial* in $e^{-j\Omega}$.
- Thus, H is a *rational function* in the variable $e^{-j\Omega}$.

Section 10.6

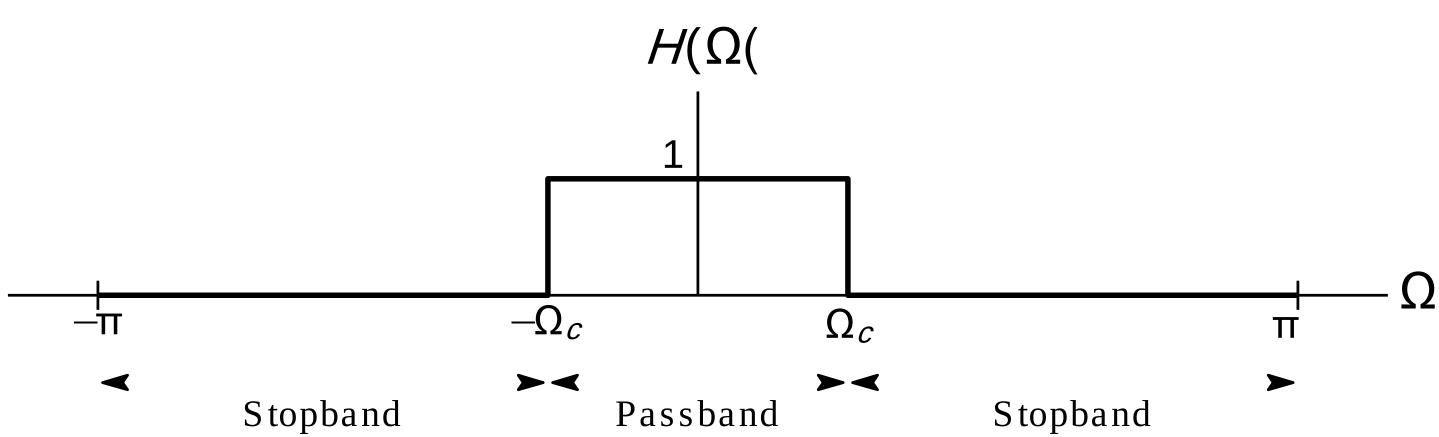
Application: Filtering

- In many applications, we want to *modify the spectrum* of a signal by either amplifying or attenuating certain frequency components.
- This process of modifying the frequency spectrum of a signal is called filtering.
- A system that performs a filtering operation is called a filter.
- Many types of filters exist.
- Frequency selective filters pass some frequencies with little or no distortion, while significantly attenuating other frequencies.
- Several basic types of frequency-selective filters include: lowpass, highpass, and bandpass.

- An ideal lowpass filter eliminates all baseband frequency components with a frequency whose magnitude is greater than some cutoff frequency, while leaving the remaining baseband frequency components unaffected.
- Such a filter has a *frequency response* H of the form

$$H(\Omega) = \begin{cases} 1 & \text{if } |\Omega| \leq \Omega_c \\ 0 & \text{if } \Omega_c < |\Omega| \leq \pi, \end{cases}$$

  where Ω_c is the cutoff frequency.
- A plot of this frequency response is given below.

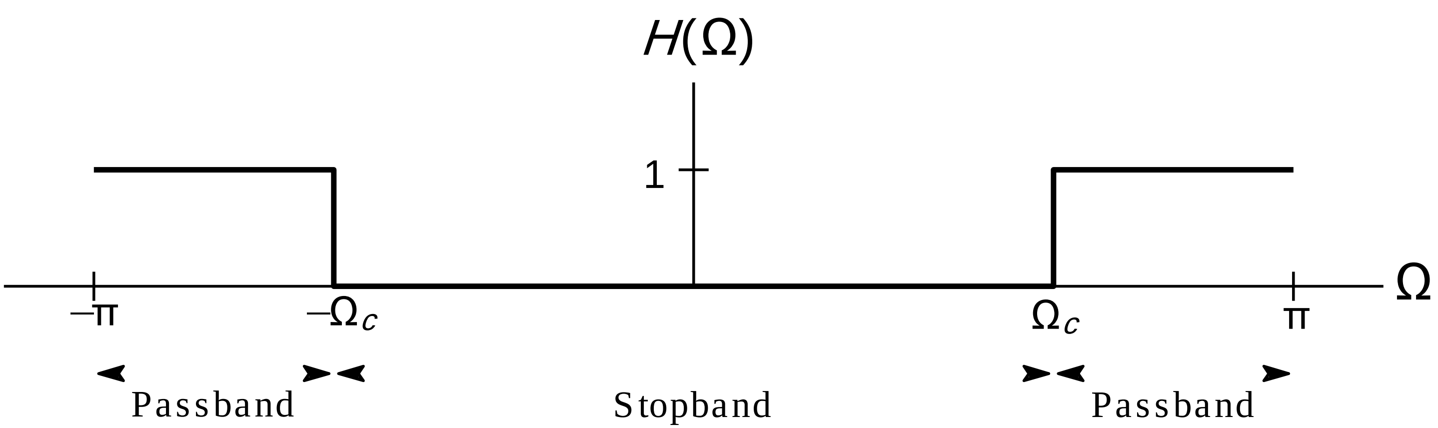


- An ideal highpass filter eliminates all baseband frequency components with a frequency whose magnitude is less than some cutoff frequency, while leaving the remaining baseband frequency components unaffected. Such a
- filter has a *frequency response* H of the form

$$H(\Omega) = \begin{cases} 1 & \text{if } \Omega_c < |\Omega| \leq \pi \\ 0 & \text{if } |\Omega| \leq \Omega_c, \end{cases}$$

where Ω_c is the cutoff frequency.

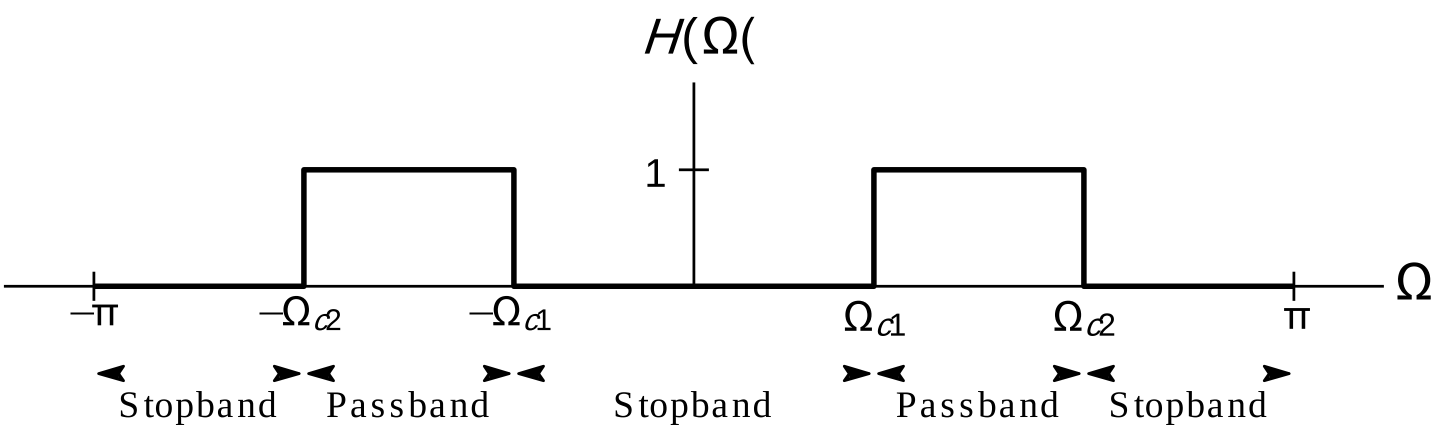
- A plot of this frequency response is given below.



- An ideal bandpass filter eliminates all baseband frequency components with a frequency whose magnitude does not lie in a particular range, while leaving the remaining baseband frequency components unaffected.
- Such a filter has a *frequency response* H of the form

$$H(\Omega) = \begin{cases} 1 & \text{if } \Omega_{c1} \le |\Omega| \le \Omega_{c2} \\ 0 & \text{if } |\Omega| < \Omega_{c1} \text{ or } \Omega_{c2} < |\Omega| < \pi, \end{cases}$$

  where the limits of the passband are Ω_{c1} and Ω_{c2}.
- A plot of this frequency response is given below.



Part 11

Z Transform (ZT)

- Another important mathematical tool in the study of signals and systems is known as the z transform.
- The z transform can be viewed as a *generalization of the Fourier transform*.
- Due to its more general nature, the z transform has a number of *advantages* over the Fourier transform.
- First, the z transform representation exists for some signals that do not have Fourier transform representations. So, we can handle a *larger class of signals* with the z transform.
- Second, since the z transform is a more general tool, it can provide *additional insights* beyond those facilitated by the Fourier transform.

- Earlier, we saw that complex exponentials are eigensequences of LTI systems.
- In particular, for a LTI system H with impulse response h, we have that

$$H\{z^n\} = H(z)z^n \quad \text{where} \quad H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}.$$

- Previously, we referred to H as the system function.
- As it turns out, H is the z transform of h.
- Since the z transform has already appeared earlier in the context of LTI systems, it is clearly a useful tool.
- Furthermore, as we will see, the z transform has many additional uses.

Section 11.1

Z Transform

- The (bilateral) z transform of the sequence x, denoted $\mathcal{Z}\{x\}$ or X, is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}.$$

- The inverse z transform is then given by

$$x(n) = \frac{1}{2\pi j} \oint_{\Gamma} X(z) z^{n-1} dz,$$

where Γ is a counterclockwise closed circular contour centered at the origin and with radius r such that Γ is in the ROC of X.

- We refer to x and X as a z transform pair and denote this relationship as

$$x(n) \overset{\text{ZT}}{\longleftrightarrow} X(z).$$

- In practice, we do not usually compute the inverse z transform by directly using the formula from above. Instead, we resort to other means (to be discussed later.(

- Two different versions of the z transform are commonly used:
  1. the *bilateral* (or *two-sided*) z transform; and
  2. the *unilateral* (or *one-sided*) z transform.
- The unilateral z transform is most frequently used to solve systems of linear difference equations with nonzero initial conditions.
- As it turns out, the only difference between the definitions of the bilateral and unilateral z transforms is in the *lower limit of summation*.
- In the bilateral case, the lower limit is $-\infty$, whereas in the unilateral case, the lower limit is 0.
- For the most part, we will focus our attention primarily on the bilateral z transform.
- We will, however, briefly introduce the unilateral z transform as a tool for solving difference equations.
- Unless otherwise noted, all subsequent references to the z transform should be understood to mean *bilateral* z transform.

- Let X and X_F denote the z and (DT) Fourier transforms of x, respectively.
- The function $X(z)$ evaluated at $z = e^{j\Omega}$ (where Ω is real) yields $X_F(\Omega)$. That is,

$$X(z)\big|_{z=e^{j\Omega}} = X_F(\Omega).$$

- Due to the preceding relationship, the Fourier transform of x is sometimes written as $X(e^{j\Omega})$.
- The function $X(z)$ evaluated at an arbitrary complex value $z = re^{j\Omega}$ (where $r = |z|$ and $\Omega = \arg z$) can also be expressed in terms of a Fourier transform involving x. In particular, we have

$$X(re^{j\Omega}) = X'_F(\Omega),$$

where X'_F is the (DT) Fourier transform of $x'(n) = r^{-n}x(n).$

- So, in general, the z transform of x is the Fourier transform of an exponentially-weighted version of x.
- Due to this weighting, the z transform of a sequence may exist when the Fourier transform of the same sequence does not.